Persistence is Trivial

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Abstract

This paper is about a conflict between two broad classes of theories about the indicative conditional. On the entailment picture, conditionals express a (potentially sui generis) entailment between antecedent and consequent. On the restrictor picture, conditionals shift the interpretation of material in their consequents. Both pictures seem to capture an important insight but, I argue, the space of tenable theories implementing both, at least to some degree, is very small. I regiment the restrictor picture as a commitment to the principle I call Persistence, the principle which says that $A > (C > A)$ is a logical truth. I then prove a range of triviality results showing that fully general versions of the restrictor picture are not cotenable with very minimal implementations of the entailment picture, and *vice versa*. I give a theory of the conditional based on the notion of acceptance and argue that it achieves the best balance of features from both pictures.

Different uses of the indicative conditional motivate different theoretical pictures of its semantics. A bare indicative seems to say its antecedent leads to or entails its consequent, albeit in some proprietary sense. Suppose Alice says to Billy:

(1) If we cut the blue wire, the bomb will be disarmed.

Alice is saying that, from the claim that you cut the blue wire, it in some sense follows that the bomb is disarmed. Call this the *entailment* picture. Now suppose Billy demurs:

(2) Well, if the red light is on, then if we cut the blue wire, the bomb will be disarmed.

Right-nested conditionals motivate the *restrictor* picture, which says that the antecedent *restricts* the interpretation of consequent:[1] in the context of Billy's utterance, the conditional *if you cut the blue wire, the bomb will be disarmed* is reinterpreted to only quantify over possibilities where the red light is on.

[1]This terminology goes back at least to Kratzer (1977, 2012). My usage of the term is more inclusive, as it seems to me Kratzer's theory is just one way to implement the more general idea. If the reader prefers, they can instead call it the information-sensitive picture.

Both pictures offer some important insight about conditionals, but there are already signs they are hard to maintain together. My aim is to investigate the space of theories that implement both insights, at least to some degree. I argue the tension is very deep: on pain of triviality, fully general versions of the restrictor picture are incompatible with very minimal implementations of the entailment picture, and *vice versa*. The insights of both pictures are more limited than they might have appeared.

To show this, I first regiment the restrictor picture as a commitment to a logical principle I call *Persistence*, the principle that $A > (C > A)$ is a logical truth. I then prove a number of triviality results: full Persistence, together with a very weak implementation of the entailment view, entails the conditional has some of the absurd properties of the material conditional analysis. These results considerably strengthen existing Import-Export triviality results against full restrictor views which, I argue, rely on more substantial implementations of both pictures. I conclude the costs of Persistence are too high for it to be valid in full generality.

Nonetheless, a weak form of Persistence, Boolean Persistence, seems extremely plausible and is not obviously impugned by these results. But when we try to add Boolean Persistence to the entailment view, I show the space is still surprisingly constrained: I prove a triviality result showing that this form of Persistence cannot be added to forms of the entailment view containing the CSO principle, a principle which the leading implementations do validate. In this case, I argue that the entailment view is at fault: Boolean Persistence is much more plausible than the instances of CSO required for this argument.

The true theory of the conditional, I argue, lies somewhere in the middle. I suggest we built a theory of the conditional on Yalcin (2007)'s notion of *acceptance*. Put roughly, an information state i *accepts* A just in case A is true throughout i, when all accessibility relations for any modal expressions in A are shifted to ones that only range over worlds in i. This notion is naturally coupled with conditionals: evaluating $A > C$ involves moving some information state that accepts A. I implement this idea in a variably strict theory of conditionals. I argue this theory captures the most plausible parts of both the entailment and the restrictor pictures, while giving satisfying explanations of where both pictures fail. This turns out to be exactly what is needed to block the triviality results.

1 Two Pictures of the Indicative Conditional

I first add more detail to our two pictures, just enough to convey the spirit shared by their implementations and to start seeing the tension between the two.

On the entailment picture, the indicative expresses a certain sui generis kind of entailment: for $A > C$ to be true is for A *in some sense* to entail C. The entailment picture is a very old one, with many of the classic theories of the indicative amounting to different articulations of the "indicative" entailment relation. On a traditional strict conditional view, the relevant notion is simply classical entailment given our evidence: $A > C$ says that the conjunction of one's evidence with A entails C. On a traditional variably strict view, the relevant notion of entailment stated by appeal to closeness: $A > C$ says that the closest worlds where A holds entails C.[2]

In fact, there is a common core shared by the leading implementations: they build on the B conditional logic, which we will study in detail in §3. There is an intimate connection between this logic and the entailment picture. Non-monotonic logic studies defeasible inference: from the premise that Tweety is a bird, we infer Tweety flies; but we of course cease to infer this on discovering Tweety is a penguin. It is natural to think that conditionals express commitments to these kinds of inferences: a belief that $A > C$ requires a willingness to infer C from A. If so, we would expect the logic of the conditional to share in the logic of this kind of inference. And indeed, many theories agree that this is so. A minimal logic of defeasible inference is the P logic of Kraus et al. (1990); and the result of translating P into conditional form is the B logic.

On the restrictor picture, through some semantic mechanism, antecedents shift the body of information held fixed by the consequent, adding to it the antecedent of the conditional. This idea can be implemented in a variety of frameworks.[3] To get a more general sense of the picture, consider some examples:[4]

(3) If the die landed on a number between 1 and 3, then if it landed on an even number, it landed on 2.

(4) If a Republican wins, then if Reagan doesn't win, Anderson will.

(3) seems obvious: the conditional in the consequent holds fixed that the die landed between 1 and 3. (4) is a truism, at least to psephophiles: the consequent holds fixed that a Republican wins; and since Reagan and Anderson were the two Republicans that year, the whole conditional is true. According to the restrictor picture, this effect is part of the semantics of the conditional: the information held fixed is *shifted* by the conditional; that is why the

[2] The classic strict and variably strict accounts here are those of Lewis (1912) and Stalnaker (1975, 2014), respectively. See also the more recent accounts of Rothschild (2011), Mandelkern (2021, forthcoming). Arguably, certain implementations of the Ramsey test conditional belong in this category also; especially those that validate the B logic mentioned below.

[3] Important examples of this kind of view include Kratzer (1977, 2012)'s restrictor account, McGee (1985, 1989)'s variation on Stalnaker's semantics, various dynamic accounts of indicative conditionals, like Gillies (2009) and Willer (2017), and the informational accounts of Yalcin (2007) and Kolodny and MacFarlane (2010); interesting recent variations include the theories in Cariani (2019, 2021), Santorio (2022), Goldstein and Santorio (2021) and Ciardelli (2021).

[4] (3) is from Goldstein and Santorio (2021) and (4) is from McGee (1985).

consequent conditional holds fixed the antecedent.

Restrictor behaviour is in evidence too when we consider epistemic modals in the consequents of conditionals. Consider:[5]

(5) If that is Venice Beach, then we can't be in Kansas.

(6) If that is Venice Beach, then we might be near LAX.

In both the antecedent restricts the domain of the modal. For instance, (5) makes no commitments about what our evidence *unconditionally* entails; rather it says that in all worlds compatible with what we know *and where we are near Venice Beach*, we are not in Kansas. Again, the restrictor picture says that it is the semantics of the conditional that shifts the modal's domain.

It is not immediately obvious that these pictures are in competition. But a tension emerges when we start to spell out the entailment picture. If the indicative expresses a kind of entailment relation, then it's natural to expect it to mirror central structural features of the notion of consequence we rely on in ordinary, deductive reasoning. An extremely simple, fundamental principle of entailment is that it is *truth*-preserving: if A entails B, then if A is true, B is true. For conditionals, this is realised in the form of Modus Ponens: if A "indicatively" entails B, then if A is true, so is B.

But Modus Ponens does not sit well with the picture of conditionals as restrictors. Return to McGee's example:

(4) If a Republican wins, then if Reagan doesn't win, Anderson will.

The restrictor view says that, in a certain sense, the consequent of this conditional is interpreted differently, when thus emdedded: there the conditional "if Reagan doesn't win, Anderson will" is interpreted as holding fixed that a Republican will win; but an unembedded assertion of the same sentence does no such thing. We would thus expect that restrictor views will tend to reject Modus Ponens: when interpreted as holding fixed that a Republican wins, the conditional "if Reagan doesn't win, Anderson will" is a truism; otherwise it is contingent and indeed highly unlikely.

The tension remains when we get more precise. I take this to be the moral of the literature on a certain class of *triviality* results, those starting with Dale (1974) and Gibbard (1981) and recently deepend by Mandelkern (2021). These results show there is very little logical space to maintain together various principles drawn from both approaches: combining important elements of both pictures tends to collapse the indicative into the material conditional. Fol-

[5] See von Fintel, Kai and Irene Heim (2021) for further argument.

lowing the vast majority, I regard this as a reductio of any view of the indicative.[6]

2 Persistence

How much of both pictures can be maintained? To answer that question, we first need some way to regiment the restrictor picture.

The challenge is that there is little semantic unity here: restrictor theories are given in many different semantic frameworks. Thus I propose to regiment the view in terms of its *logic*. I take a central commitment of a full restrictor picture to be the principle I call *Persistence*.[7]

Persistence. $A > (C > A)$

More narrowly, we can say that > is a restrictor with respect to constructions A and C if it validates Persistence for those constructions.[8]

Persistence is simple, hopefully even truistic. Attempts to deny the principle or hedge on its truth simply seem confused:

(7) #If it's raining, then it's not the case that if it is cold, then it is raining.

(8) #If it's raining, then if it's cold, then maybe it's not raining.

But Persistence also captures the intuitive spirit of the restrictor picture: a right-nested conditional holds fixed the initial antecedent. It thus explains the patterns exemplifying the restrictor view. Return to the McGee example, (4). Persistence tells us that the following is valid.

(9) If a Republican wins, then if Reagan doesn't win, a Republican wins.

The following is also plausibly valid:

(10) If a Republican wins, then if Reagan doesn't win, Reagan doesn't win.

And together, (9) and (10) entail (4).

Persistence also explains the epistemic modal data, given independent relationships between epistemic modals and conditionals. Following Dorr and Hawthorne (ms.), it is natural

[6] For futher catalogues of the usual arguments, see Gillies (2012) and Boylan and Schultheis (2022); though see also Williamson (2020) for a recent defence of the material analysis.

[7] A word on notation. I use > for the indicative conditional; I use upper case letters for sentences of any degree of complexity; and I use lower case letters for atomics.

[8] As we'll see, few views succeed in fully vindicating the restrictor picture.

to identify epistemic necessity claims with certain conditional claims. We might take *must* A to simply be equivalent to $\neg A > \bot$:

Identity of Accessibility. *must* $A \leftrightarrow (\neg A > \bot)$

Standardly, $\neg A > \top$ is true only when its antecedent is in some sense impossible. Identity of Accessibility stipulates that this sense is epistemic. As Dorr and Hawthorne note, this would explain various inferences moving between epistemic modals and conditionals, Take for instance, Must Preservation:

Must Preservation. $($*must* $C) \supset (A > C)$

This pattern appears valid:

(11) a. The car can't be working properly.
b. So, (even) if the car starts, it isn't working properly.

This trivially follows from Identity of Acccessibility.

Given a weak background logic, Persistence entails a principle I call Modal Persistence:[9]

Modal Persistence. $A >$ *must* A

Given Identity of Accessibility, Modal Persistence explains the restrictor data for epistemic modals. Recall:

(5) If we are near Venice Beach, then we must not be in Kansas.

Modal Persistence gives us:

(12) If we are near Venice Beach, then we must be near Venice Beach.

And if it's epistemically necessary we're near Venice Beach, then it's epistemically necessary that we're not in Kansas.

There is an existing principle one might think already captures the restrictor view, namely Import-Export. Import-Export is the following principle:[10]

Import-Export. $(A \wedge B) > C \leftrightarrow A > (B > C)$

[9]Proof sketch: assume Persistence; this gives us $A > (\neg A > A)$; from Identity and CA, we have $A > (\neg A > \neg A)$; using CA again we can get $A > (\neg A > \bot)$ and from Identity of Accessibility we get $A >$ *must* A. Adding MOD, introduced in §2, makes Persistence and Modal Persistence equivalent.

[10]I use $\leftrightarrow$ for the material biconditional.

Besides concerning right-nested conditionals, there are obvious affinities between Import-Export and Persistence. Import-Export is validated by many restrictor type views.[11] And Import-Export could also be used to explain the data we have observed.

Nonetheless, I submit that Persistence provides a better precisification of the restrictor view. Import-Export is a strictly stronger principle, given the weak background logic I will assume in §2. It's relatively easy to derive Persistence from Import-Export in this setting: Identity and Consequent Closure give us $(A \wedge C) > A$ and Import-Export then gives us Persistence. In Fact 1 of Appendix A, I show that Import-Export is strictly stronger. The basic reason why is simple. Import-Export imposes a relationship between iterated updates and certain one-off updates: updating with A and then B must be the same as updating with $A \wedge B$. This suffices to derive Persistence, given a weak background logic, but it is not required: Persistence itself is silent about the relationship between iterated updates and any one-off updates. This further commitment of Import-Export does not obviously have anything to do with the restrictor picture. (That being said, I will in §4.2 compare my triviality results to some of the main Import-Export triviality results)

3 Triviality

Persistence is in tension with a very minimal way of spelling out the entailment picture, one where the conditional has some basic features of *classical* entailment. I'll first outline three particular assumptions motivated by this thought. Then I derive three triviality results from those assumptions and Persistence.

3.1 The assumptions

My first assumption is Identity:

Identity. $A > A$

My second assumption is that conditional consequents agglomerate:

Consequent Agglomeration. if $B_1, \dots, B_n \vdash C$ then $A > B_1, \dots, A > B_n \vdash A > C$

Classical consequence of course obeys this principle: if A entails $B_1, \dots B_n$ which in turn entail C, then A entails C. My final assumption is the MOD principle:

MOD. $(C > \bot) \supset (A > \neg C)$

[11] For non-conditional antecedents at least. In fact the number of views that *fully* validate Import-Export is actually quite small; see Mandelkern (2018) for discussion.

Roughly, MOD says that if C indicatively entails $\perp$, then everything indicatively entails $\neg C$.

All three principles are naturally taken to be part of the entailment picture. Consider them individually. Identity says, very plausibly, that like most notions of entailment "indicative" entailment is reflexive.[12] Consequent Agglomeration corresponds to the thought that indicative entailment should be closed under classical entailment. Finally, MOD corresponds also to a feature of classical entailment, the principle that something contradictory is always entailed to be false: if A entails $\perp$ then any B entails $\neg A$.

There is also a further reason to regard these principles as part of the entailment picture, namely, that they are part of B, the basic conditional logic mentioned in §1. B, formulated in Burgess (1981), results from adding to Consequence Agglomeration and Identity the rule of LLE and the axioms of CSO and OR:

LLE. If $\vdash A \leftrightarrow B$ then $\vdash (A > C) \supset (B > C)$

CSO. $(((A > B) \wedge (B > A)) \wedge (A > B)) \supset (B > C)$

OR. $((A > C) \wedge (B > C)) \supset ((A \vee B) > C)$

MOD is also a theorem of this system.[13] As mentioned B seems intimately connected to the entailment picture; the leading implementations are committed to all of our assumptions.

In the arguments that follow, I assume our logic is propositional logic plus Identity, Consequent Agglomeration, MOD and Persistence:[14] any instance of a PL theorem is a theorem. It will be helpful to note two theorems of this system I will appeal to throughout. I will often cite the single premise case of Agglomeration, which I call *Consequent Closure*:

Consequent Closure. If $B \vdash C$ then $A > B \vdash A > C$.

Finally, I will sometimes appeal to the principle I call Normality:

Normality. $A > \top$

This follows trivially from Identity and Consequent Closure.

[12] Though Klinedinst and Rothschild (2014) and Mandelkern (2020) note that update-to-test notions of consequence do not necessarily have this feature.

[13] Proof sketch. Assume $\neg C > \perp$. Using Consequent Closure we derive $\neg C > A$ and $\neg C > C$. From these two, LLE and Cautious Monotonicity we can derive $(A \wedge \neg C) > C$. From Identity and CC we already have $(A \wedge C) > C$. Applying OR, we can then derive $A > C$.

[14] Note that some respond to triviality by giving up this assumption: Gillies (2009) in particular denies that $\neg A$ and $\neg B$ are necessarily logically equivalent, whenever A and B are equivalent. This will not help in replying to Triviality 2 or 3.

3.2 First triviality result

Identity, Persistence, Consequent Agglomeration and MOD taken together allow us to prove that in the consequent of a conditional, a material conditional always materially entails the corresponding indicative.

Triviality 1. $B > ((A \supset C) \supset (A > C))$

The basic proof strategy is this. We can use Identity and CA to derive:

A. $((A \supset C) \wedge \neg(A > C)) > \neg(A > C)$

We can use Persistence and CA to derive:

B. $((A \supset C) \wedge \neg(A > C)) > (A > (A \supset C))$

Given B, Identity and CA allow us to derive.

C. $((A \supset C) \wedge \neg(A > C)) > (A > C))$

But Triviality follows from C, given MOD and CA.

This cannot be accepted, as it pushes the logic of the indicative far too close to the logic of the material. It is easy to see that, given weak additional assumptions, Triviality 1 makes material conditionals straightforwardly entail indicatives. To start with, simply substitute any tautology in the antecedent of Triviality 1:

$$\top > ((A \supset C) \supset (A > C))$$

It is extremely natural to think that if $\top > C$ holds, then C itself must hold. This is an extremely weak form of Modus Ponens, which we might call Tautologous Modus Ponens:

Tautologous MP. $(\top > C) \supset C$

Even without Tautologous Modus Ponens, the theory of indicatives is *still* too close to the material conditional. Triviality 1 says that right-nested conditionals are entailed by material conditionals. This reintroduces the paradoxes of material implication, specifically in the consequents of conditionals. For instance in the consequent of a conditional, $\neg q$ entails $q > r$:

$$p > \neg q \models p > (q > r)$$

This is no more plausible than the principle that $\neg q$ entails $q > r$:

(13) a. If it rains, it won't be cold.

b. ??So, if it rains, then if it's cold, there will be a picnic.

Even worse, a negated conditional $\neg(q > r)$ nested in the consequent of a conditional entails $q \wedge \neg r$:

$$p > \neg(q > r) \models p > (q \wedge \neg r)$$

Again, this is no more plausible than the claim that materials straightforwardly entail indicatives:

(14) a. If it rains, it's not the case that if it's warm there will be a picnic.

b. ??So, if it rains, it will be warm and there won't be a picnic.

We should reject these results: materials do not entail indicatives, not even in the consequents of indicatives.

3.3 Second triviality result

Next we prove the following.

Triviality 2. $\neg(A > C) > (B > \neg C)$

This says that, given a negated indicative, *any* further supposition entails the negation of the consequent.

Here is a proof sketch. Given Persistence, $C > (A > C)$ is a logical truth; so, for any antecedent B whatsoever, $B > C > (A > C)$ holds. Letting B be $\neg(A > C)$ we get $\neg(A > C) > C > (A > C)$. But Persistence also gives us $\neg(A > C) > C > \neg(A > C)$, allowing us to derive $\neg(A > C) > C > \bot$. From MOD, we derive $\neg(A > C) > B > \neg C$.

Triviality 2 also makes the logic of the indicative far too close to that of the material. Sentences of this form are not at all tautologous:

(15) If it's not the case that there will be a picnic if it rains, then if my favourite number is seven, then there won't be a picnic.

(16) If it's not the case that there will be a picnic if it rains, then if God exists, then there won't be a picnic.

Remember that $\neg(A > C)$ does not entail that $\neg C$: we may take it to be false that there will be a picnic if it rains, and still allow there may be a picnic. Supposing some further, unrelated proposition makes no difference.

Worse still, Triviality 2 offers various ways of deriving $\neg(A > C) > \neg C$. One could substitute $\top$ for B in Triviality 2 and then use Tautologous MP and Consequent Agglomeration to derive $\neg(A > C) > \neg C$. Alternatively, consider the principle Contraction:[15]

Contraction. $(A > (A > C)) \supset (A > C)$

Contraction seems extremely plausible: iterating the same antecedent is indeed idle. But Contraction also allows us to derive $\neg(A > C) > \neg C$ from Triviality 2: simply substitute $\neg(A > C)$ itself for B.[16]

3.4 Third triviality result

Lastly, we prove:

Triviality 3. $\neg(A > B) > ((\neg A > C) > \bot)$

This is perhaps easier to grasp in suppositional terms: supposing two negated conditionals, where the antecedent of one is the negation of the antecedent of the other, leads one into a contradictory state.

Here is a proof sketch. From Persistence and CA we can prove:

A. $\neg(\neg A > C) > (A > (\neg A > \bot))$

But in general, $\neg(A > C)$ entails $\neg(A > \bot)$, given CA. So from Persistence and CA we can also prove:

B. $\neg(\neg A > C) > (A > \neg(\neg A > \bot))$

Applying Agglomeration to A and B we derive:

C. $\neg(\neg A > C) > (A > \bot)$

Then using the principle that if $\vdash C$ then $\vdash A > C$ gives us:

D. $\neg(A > B) > (\neg(\neg A > C) > (A > \bot))$

However, from another instance of Persistence, plus the fact that $\neg(A > C)$ entails $\neg(A > \bot)$, we also have:

[15] I take this name from Bonevac et al. (2006).

[16] While Contraction is entailed by Identity and Modus Ponens, the frame in the proof of Fact 1 shows it can hold in the absence of Modus Ponens.

E. $\neg(A > B) > (\neg(\neg A > C) > \neg(A > \bot))$

We get Triviality 3 by applying CA to D and E.

To why Triviality 3 must be resisted, think first about it in more theoretical terms: Triviality 3 says that supposing in sequence some negated conditional $\neg(A > B)$ followed by $\neg(\neg A > C)$ leads to inconsistency. But why would this be? It seems perfectly consistent to assert both:

(17) It's not the case that if it rains there will be a picnic and it's not the case that if it doesn't rain, there won't be a picnic.

Why would *supposing* these in sequence be any worse? The material theory naturally offers an explanation: the initial antecedent entails A and the second antecedent entails $\neg A$; so revising with these two conditionals involves revising with inconsistent antecedents. But this is exactly one of the material theory's bad predictions.

Another way to dramatise the problem is by appeal to Identity of Accessibility. Given the duality of *must* and *might*, that principle tells us that $\Diamond A$ is equivalent to $\neg(A > \bot)$. One instance of Triviality 3 is:

(18) $\neg(A > \bot) > (\neg(\neg A > \bot) > \bot)$

Then applying Identity of Accessibility gives us:

(19) $\Diamond A > (\Diamond\neg A > \bot)$

This is a particularly absurd result. $\Diamond A$ and $\Diamond\neg A$ are not inconsistent; so why would supposing them jointly lead to inconsistency? Furthermore there seem like pretty natural, assertable examples of conditionals with the form $\Diamond A > (\Diamond\neg A > C)$. Consider:[17]

(20) If Bob might be in his office then if Bob might not be in his office, then we don't know whether Bob's in his office.

Finally, we can directly consider instances of Triviality 3. Take:

(21) If it's not the case that there will be a picnic if it rains, then if it's not the case that there will be a picnic, if it *doesn't* rain, then there will be a picnic.

Triviality 3 suggests such a conditional should be trivially true. This seems wrong: it is a contingent, and likely dubious, piece of information. Those who like CEM should be especially

[17] Thanks to [XXX] for this example.

wary here. For them, when both antecedents are possible, $\neg(A > C)$ and $\neg(\neg A > D)$ are equivalent to $A > \neg C$ and $\neg A > \neg D$. Thus, when it is possible that it will rain and possible that it will not, (21) above should be equivalent to:

(22) If there won't be a picnic if it rains, then if there won't be a picnic if it doesn't rain, then there will be a picnic.

Again this is contingent and dubious, not trivially true.

4 Significance of these results

We have now ample reason to see that there is a conflict between Persistence and the entailment view, as I have spelled it out.

In this section, I first argue these results show us that Persistence, and so in turn the restrictor picture, cannot be maintained in full generality. Second, I compare my results to the existing triviality results for Import-Export, showing that they significantly strengthen those of Gibbard (1981) and Mandelkern (2021).

4.1 Against Persistence

Abandoning Consequent Agglomeration is, to my mind, a no-go. How could one deny that (23) and (24) entail (25)?

(23) If there is a party, Alice will come.

(24) If there is a party, Billy will come.

(25) If there is a party, Alice and Billy will come.

Rejecting this is not obviously better than falling into triviality. And my uses of this principle are not applied to a large set of premises, where we might just be able to stomach a failure of Agglomeration.[18]

One might consider rejecting Identity. Here there is more theoretical space, for as Mandelkern (2021) has shown, a surprisingly large class of theories give up Identity; indeed, some such as Cariani (2019, 2021) have endorsed giving up certain "junk" instances of Identity in order to maintain Import-Export. Such junk instances tend to involve complex, left nested conditionals, examples which are indeed hard to parse. However, this strategy does not seem a viable escape route to me here.

[18] See for instance Leitgeb (2012), where Agglomeration fails precisely because high probability does not agglomerate.

First, rejecting Identity while accepting Persistence is an unstable combination. If Identity fails then, for some A, $A > A$ can be false. But if Persistence holds, then *for the very same* A, $A > (C > A)$ is valid; indeed $A > (\top > A)$ is valid, even though $A > A$ is not. I cannot imagine assenting to $A > (\top > A)$ while dissenting from $A > A$.

Second, the role of Identity is extremely minimal in my second and third arguments. In my second argument, Identity itself is not essential. In fact, Normality would suffice: at step 2 we could appeal to $\neg(A > C) > \top$. Really we would need to deny *any* conditional of the form $\neg(A > C) > B$ is logically true, if we were to block the argument at this step. In my third argument, Identity does indeed seem necessary: to get to step 7, we need both $\neg A > (A > \neg A)$ and $\neg A > (A > A)$, the latter of which is derived using Identity. But this is not one of the junk instances of Identity mentioned above.

So the only option left to the defender of Persistence is to reject MOD. But this is not enough to rescue the full strength of Persistence because in fact MOD is not involved in our last triviality argument: Triviality 3 in fact only requires Identity, Consequent Agglomeration and Persistence to prove. So, even while we will see I think there is a principled way to reject MOD, this move by itself is not enough to get out of the triviality results.

4.2 Comparison to existing results

Before moving on, it is worth briefly touching on the existing triviality arguments for Import-Export. My arguments strengthen two of the most important existing results, those of Gibbard (1981) and Mandelkern (2021).

First, my results illustrate is that the full strength of Import-Export is not necessary to generate triviality. We aleady saw that, given the logic in §3, Import-Export is strictly stronger than Persistence. Indeed, I think a plausible diagnosis is that Import-Export leads to triviality precisely *because* Import-Export entails Persistence in such a setting.

Second, my third result closes off various responses one might have given to existing results. Gibbard proved a collapse to the material from Import-Export, Modus Ponens and Logical Implication:

Logical Implication. If $A \models C$ then $\models A > C$

Mandelkern (2021) proves a collapse to the material using Identity, Import-Export and the following two principles:

Very Weak Monotonicity. If $\models A > A$ then if $A \models B$ then $\models A > B$

Ad Falsum. $A > B, A > \neg B \models \neg A$

Both of these results rely on assumptions that many will reject. McGee (1985) produced prima facie counterexamples for exactly the instance of Modus Ponens used in Gibbard's result. Mandelkern's Ad Falsum will not obviously seem acceptable to those who reject Modus Ponens: after all, it is an instance of Modus Tollens.[19] My third result, relying only on Persistence, Identity and Consequent Agglomeration, shows we cannot avoid triviality by denying these assumptions.

Finally, prima facie, the ancillary assumptions of Triviality 1 and 2 are on stronger footing than those of Gibbard and Mandelkern. The crucial extra assumption for Triviality 1 and 2 is MOD. But it is not at all clear that the denier of Modus Ponens should deny MOD: unlike Ad Falsum, it is not an instance of any principle intimately related to Modus Ponens.

Could MOD nonethless be more vulnerable than the assumptions in the other results? No, as those who accept Gibbard or Mandelkern's assumptions are more or less committed to MOD. Gibbard's assumptions directly entail it: given classical logic, Modus Ponens entails a principle I'll call Contradiction:[20]

Contradiction. $(A > \bot) \supset \neg A$

This principle generates MOD.[21] Mandelkern's Ad Falsum is also clearly very close to Contradiction: it is essentially the conjunction of Contradiction with an instance of CA, namely that $A > \bot$ entails $\neg A$; indeed, I don't see how one could maintain Ad Falsum without Contradiction.

5 Against the entailment picture

At this point, the most natural package would seem to be one that combines the entailment view with a more limited version of the restrictor picture, one where Persistence is limited to Boolean antecedents. After explaining why I take that package to be the best motivated so far, I will show that it also succumbs to its own triviality result.

[19] What's more it is invalidated by a range of dynamic and acceptance conditionals. As we will see in §7, acceptance conditionals invalidate MOD for antecedents like $A \supset C \wedge \neg(A > C)$. For basically the same reasons, Ad Falsum too will fail in these frameworks.

[20] Proof sketch: Suppose $A > \bot$ and for contradiction suppose A. Then from Modus Ponens $\bot$ follows. Classical reasoning then gives us that $\neg A$ must hold.

[21] Proof sketch. Assume $\neg A > \bot$; from Consequent Closure we get $\neg A > (C > \bot)$; two applications of Import-Export yield $C > (\neg A > \bot)$; finally, Contradiction and Consequent Closure yield $C > A$. Note that Fact 3 in Appendix A shows that without Modus Ponens, the remaining assumptions do not entail Contradiction.

5.1 The trouble with Boolean Persistence

So far, no single part of the entailment view has been brought into question. The only principle used in all three results is non-negotiable, Consequent Agglomeration; whereas Identity is not really required in the second result and MOD is not necessary for the third result. But various instances of Persistence seem very much up for grabs. All of our triviality results involved conditionals with complex, left-nested antecedents. The following are required in Triviality 1,2 and 3, respectively:

$$(A \supset C) \wedge \neg(A > C) > (A > ((A > C) \wedge \neg(A \supset C))$$

$$\neg(A > C) > (C > \neg(A > C))$$

$$\neg(A > C) > (\neg A > (\neg A > C))$$

These structures are distant from the simple Boolean examples with which we motivated the restrictor picture. Indeed left nested conditionals are so hard to process that one might think full Persistence is unlikely to be needed to account for any data.

A very natural response then is keep all of the entailment picture and to accept a limited form of Persistence, namely for just Boolean antecedents:

Boolean Persistence. $A > (C > A)$, when A, C are Boolean.

That way, we do justice to the original motivating examples, without falling afoul of our earlier results. It might seem that we are also able to maintain all of the entailment picture.

But in fact even Boolean Persistence is in tension with a basic principle in conditional logic, one that also looks like it should form part of the entailment picture, namely CSO:

CSO. $((A > B \wedge B > A) \wedge A > C) \supset B > C$

As with our other principles, there are two ways to see why this belongs in the entailment picture. First, consider simply the content of the principle: phrased in terms of entailment, CSO says if A and B conditionally entail each other, then A and B conditionally entail the same things. This is an extremely natural property of consequence relations. Secondly, CSO is also part of the B conditional logic.

CSO, however, does not play well even with Boolean Persistence it turns out. To see this, start with a case. Suppose that, in the upcoming US election, the third party is overwhelmingly likely to win: they have about a 90% chance of victory, with the Democrat and Republican candidates having each a mere 5% chance. In this case, the following are extremely probable:

(26) If the Democrat or the third party candidate wins, it will be the third party candidate.

(27) If the Republican or the third party candidate wins, it will be the third party candidate.

These should strike us as having over 90% chance of being true. But (26) and (27), together with some trivial conditional logic, give us that the following are each at least as likely:

(28) If the Democrat or the third party candidate wins, the third party candidate or the Republican wins.

(29) If the Republican or the third party candidate wins, the third party candidate or the Democrat wins.

Now the following instance of Boolean Persistence is a truism.

(30) If the Republican loses, then if third party candidate loses, then the Democrat wins.

After all, there are just three candidates and so one of them must win. Thus, from (30), (28) and (29), CSO should allow us to infer that the following is very likely:

(31) If the Democrat loses, then if the third party candidate loses, the Democrat wins.

But of course (31) is in fact absurd.

We can prove that this tension is more general:

> Triviality 4. Suppose that A, B, C are pairwise inconsistent. Then, given CSO, Identity, Consequent Agglomeration and Boolean Persistence,
>
> $$(A \vee B) > B, (B \vee C) > B \models (A \vee B) > \neg B > \bot$$

Essentially, CSO conflicts with the restricting effect of Boolean Persistence. CSO tells us that, if it happens to be true that the Democrat loses, if the Republican does and *vice versa*, then we should be freely able to substitute "the Republican loses" with "the Democrat loses" in antecedents. Boolean Persistence says that when we consider conditionals with antecedents like "the Republican loses" or "the Democrat loses", that information should be held fixed by right-nested conditionals. But we do not want a conditional with the antecedent "the Republican loses" to *also* hold fixed that the Democrat loses. These two antecedents have *different* effects on the information held fixed by conditionals in the consequent: picking pairwise inconsistent disjuncts dramatises what the consequences of taking them to have the same effect.

Unlike our previous results, this problem seems to impugn the entailment picture and not Boolean Persistence. Our previous results appealed to complex, hard-to-assess instances of Persistence; but instances of Boolean Persistence seem on extremely solid footing. Consider again (30):

(30) If the Republican loses, then if third party candidate loses, then the Democrat wins.

This is just a truism about the set-up of the case. CSO seems far more likely to be the offender: (28), (29), (30) and (31) offer a much more prima facie compelling counterexample to CSO.[22]

5.2 Weakening Boolean Persistence

To maintain the entailment picture, we might try to weaken the status of Persistence even further. Following the inspiration of Lewis (1996), we might maintain that Boolean Persistence is invalid, but also say that, because of the pervasive context-sensitivity of conditionals, those counterexamples to Persistence are *elusive*.

On this strategy, Persistence is not valid; but when instances of Persistence are uttered, context shifts to make them true. Take again:

(30) If the Republican loses, then if the third party loses, the Democrat wins.

[22] While other counterexamples to CSO have been offered, they arguably rely on more tendentious judgements. Tichý (1976) offers a counterexample with the following structure (though Bacon (2012) attributes this version to Stalnaker):

> The crown jewels are on an open display platform surrounded by electric eye sensors. A cat is sleeping on the platform, near the jewels but outside the circle of electric eyes. If anyone, human or cat, were to reach into the dispay area an alarm would sound. If the alarm were to sound, it would wake up the cat. If the cat were to wake up, he would cross into the display area, setting off the alarm.

We are supposed to accept as true:

(i) If the alarm sounds, the cat will wake.

(ii) If the cat wakes, the alarm will sound.

(iii) If the cat wakes, he will set off the alarm.

We are supposed to reject as false:

(iv) If the alarms sounds, the cat will (have) set off the alarm. t

I am skeptical. Suppose I am very confident in (iii): then I must have very low confidence that the cat wakes as a result of the burgular setting the alarm off; otherwise why would I be so confident that it would be the cat who sets it off? But in that case, (iv) no longer seems so implausible. The same basic issue affects the counterexamples of Ahmed (2011) and Bacon (2012): once we establish what it takes for the $A > C$ conditional to be probable, the $B > C$ conditional no longer seems improbable. (But see also Walters (2011).)

The thought is that, by uttering this sentence, context shifts so that the innermost conditional holds fixed the information that a Republican loses: once it is uttered, we never interpret (30) in a way where the innermost conditional "forgets" the information that a Republican loses. The same happens when we utter the conditional:

(32) If the Democrat loses, then if the third party loses, the Republican wins.

However, the context where (32) is true may be one where (30) is *false*, and vice versa. This is the sense in which counterexamples to Persistence are elusive: after first asserting (30), then asserting (32), for example, pushes us towards an interpretation that makes it true, even if previously we were in a context where it is false. This is a common response to McGee's counterexamples to Modus Ponens, and the appeal of Import-Export; and more recently, in the form of Mandelkern (2021, forthcoming)'s *bounded* theory of indicative conditionals, we find a highly sophisticated version of this basic strategy.[23]

This view responds to our latest result by saying that (30) and (31) get evaluated in different contexts.[24] (30) gets interpreted as holding fixed the information that a Republican loses. In that context, the problematic (31), repeated below, is in fact true:

(31) If the Democrat loses, then if the third party loses, the Democrat wins.

But uttering (31) moves us into a context where the corresponding instance of Persistence, (32), is true; in that context, (31) has the false reading we hear; furthermore, (30) is false in that context. All this allows us to maintain CSO, while capturing the appearances in this case: when everything is interpreted univocally, there is no counterexample to CSO.

The problem, however, is that it seems that we can quantify over the instances of Boolean Persistence driving the problem. Consider:

(33) If a given party lost the election, then the other party won, if the third party candidate didn't win.

[23] There are some differences within these views. One option is to say that the consequent of a right-nested conditional receives a special interpretation, different from that of the main connective. For example, on this view, (30) is interpreted as follows:

(i) Republican loses > (third party loses $>_R$ Democrat wins)

Mandelkern, on the other hand, takes the conditional to be interpreted univocally; instead, when interpreting sentences like (30), the salient notion of closeness to be one where, for any given epistemically possible world where the antecedent is true, the closest worlds are all ones where the antecedent is true. I state the arguments below in general terms, as I believe both kinds of views will struggle with quantified cases.

[24] Similarly in the proof of Triviality 4, it says that the two instances of Persistence appealed to are not true in the same context.

This quantified sentence commits us to both (30) and (32) at once; we are saying:

(34) For both parties x: if party x loses then if the third party loses then x's rival party wins.

Given CSO, together with (26) and (27), (33) should entail:

(35) If a given party lost the election, then it won the election, if the third party candidate didn't win.

This is of course absurd.

Here our context-shifting strategy does not help. It falsely predicts that (33) cannot be heard as true in the first place; after all, there is not supposed to be a single context in which both of the relevant instances of Persistence are true.

Probably best thing for the elusive theorist to say here is that in (33) the conditional itself can be directly bound by the quantifier: for concreteness, we could assume the conditional comes with a bindable individual variable in its logical form; and the relevant notion of closeness would be a function of the value of this variable.[25] This would allow the elusive theorist to make (33) true. Its structure would be something like:

(36) For both parties x: (party x loses $>_x$ (the third party loses $>_x$ x's rival party wins)

Thus there would be no single interpretation of the conditional where both instances of Persistence are true: (30) is true on the reading where x is the Republican; (32) is true when x is the Democrat.

But this will disrupt the apparent validity of *other* patterns of inference that have nothing to do with with right-nested conditionals, like Agglomeration. After all, embedding simple conditionals under quantifiers will shift the relevant notion of closeness too. To see this, consider:

(37) Everyone who came to the party lost the lottery, if a ticket was drawn.

(38) Everyone who came to the party was someone who lost the lottery if a ticket was drawn.

It should now follow by Agglomeration that:

(39) If a ticket was drawn, everyone who came to the party lost the lottery.

If it's true of each person that they lost if a ticket was drawn, then if a ticket was drawn, all

[25] Thanks to [XXX] for this suggestion.

of those people lost.

On the strategy above, such inferences will not necessarily be perceived to be valid. (37) and (38) should have a reading where the quantifier binds the conditional:

(40) For every x who came to the party: a ticket was drawn $>_x$ x lost the lottery.

But it wouldn't follow that for any particular x the following is true:

(41) a ticket was drawn $>_x$ everyone who came to the party lost.

After all, each instance of (40) is about a different conditional and thus we cannot agglomerate them to derive (41). In fact, things like the following should have consistent readings, even if we suppose that tickets for the lottery were bought by all and only the people at the party:

(42) It's possible that everyone who came to the party lost, if a ticket was drawn; even though if a ticket was drawn, then somebody at the party won.

(43) It's possible that everyone who came is someone who lost, if a ticket was drawn; though of course if a ticket was drawn, then somebody at the party won.

To bring this out, note that if the lottery is big enough, then for any given person s the following conditional should seem likely:

(44) If a ticket was drawn, s lost.

All accessible interpretations of the above seem to have high probability. Now, while it should not have particularly high probability, the conjunction of the conditionals *ticket drawn* $>_s$ *s lost* should not have 0 probability either: the selected *ticket drawn*-world for each $>_s$ could be one where s loses, since for each $>_s$ we have a different notion of closeness. These are clear cases of overgeneration.

6 Acceptance conditionals

Neither of our theoretical pictures are tenable in full generality. Full Persistence leaves us with a theory far too close to the material analysis, unless we give up extremely basic principles. The full entailment view is not easy to reconcile with Boolean Persistence, a principle which seems necessary to explain the appeal of Persistence. I suggest that a good balance of both is maintained by a theory of the conditional based on the notion of *acceptance*.

6.1 The acceptance response, explained informally

Roughly, a body of information — that is, a set of worlds — accepts a sentence A when A is true relative to every world in that information state *when all accessibility relations are shifted so that only worlds in that information state are accessible.*

Acceptance has structural features that mere entailment does not. For one, acceptance is not persistent: an information state can accept A, even while a strict subset of it rejects A. For example, if my belief state is consistent with rain, then I likely will accept the claim that it might rain: when epistemic accessibility is interpreted to range over my belief worlds, *might rain* is true throughout my belief state. However, a subset of my belief state containing only ¬rain worlds will certainly not accept this claim: when epistemic accessibility is interpreted to range over only worlds in that subset, *might rain* must fail throughout. Entailment, modelled with the subset relation, of course cannot give rise to this kind of situation: anything entailed by an information state is entailed by all of its subsets.

It is straightforward to think about conditionals in terms of acceptance. Many agree that to evaluate $A > C$ we suppose A and then evaluate whether C. But what does supposing A involve? A natural answer is *moving to a state which accepts* A. This kind of approach has already proved fruitful. Goldstein and Santorio (2021) show an acceptance based semantics furnishes a novel response to dynamic triviality results for Stalnaker's Thesis. Boylan and Schultheis (2022) and Boylan (2024) show that informational approaches allow us to separate principles Stalnaker's Thesis from substantial epistemological theses like Negative Introspection or the falsity of margin for error principles.[26]

The acceptance picture preserves the best motivated parts of both the restrictor picture and the entailment picture. On the entailment side, this picture validates Identity and Agglomeration: when I update to a state that accepts A, A will of course be true throughout that state; and if updating to an A-accepting state leaves me in a state where B and C both hold, then it leaves me in a state where $B \wedge C$ holds.

The acceptance picture will also validate Boolean Persistence. On an acceptance semantics, updating with an antecedent necessarily alters the accessibility relation for subsequent conditionals. When we evaluate a conditional of the form $A > (B > C)$, we first update with A and then evaluate whether $B > C$ holds in the relevant information state. On an acceptance semantics, this update with A changes the accessibility relation we use to evaluate $(B > C)$. Unembedded, $B > C$ would not necessarily be evaluated relative to an information state that accepts A; but when right-nested in $A > (B > C)$ it does. When A is persistent, Persistence holds: updating with A will constrain the accessibility relation

[26] Dynamic approaches here are of course close cousins. A major difference, however, is that such views do not validate Identity, as their notion of update is not idempotent. Since they do not validate Persistence either, this seems to me a definite cost, as compared to the acceptance view.

for $B > C$, *even after* we update with B. Since Booleans are persistent in this framework, Boolean Persistence will be valid.

The distinctive structural features of acceptance also furnish us with responses to all of our triviality results. The first triviality result is blocked because, while valid for Booleans, MOD is not valid in full generality. Recall that MOD says:

MOD. $(C > \bot) \supset (A > \neg C)$

In other words, if updating with C leaves one in a contradictory state, then any state you like yields $\neg C$. An acceptance-based conditional will not validate MOD.

Here is one way to see this. An informational tautology is a claim accepted by every information state; and an informational contradiction is a sentence accepted by no consistent information state. Unlike their classical analogues, informational contradictions and tautologies are non-dual: even if a sentence is accepted by no consistent information state, its negation may not be accepted by every consistent information state. For example, no information state i accepts *rain* $\wedge$ *might* $\neg$*rain*. But now consider its negation, $\neg$(*rain* $\wedge$ *might* $\neg$*rain*): this is equivalent to *rain* $\supset$ $\neg$*might rain*. This is not something you accept when you are uncertain about whether it will rain: if you accept that it might rain and that it might not, then *rain* $\supset$ $\neg$*might rain* will fail at any $\neg$rain-world in your belief state.

This lack of duality forces MOD to fail. Updating with $\Diamond C \wedge \neg C$ never yields a consistent state; we already saw that it is an informational contradiction. Thus $(\Diamond C \wedge \neg C) > \bot$ will be trivially true: the only state which accepts the antecedent is inconsistent and so also accepts $\bot$. But $A > \neg(\Diamond C \wedge \neg C)$ will not in general be trivially true. For instance, let A just be $\top$. We already saw that $C \supset \neg \Diamond C$ is not accepted by states that accept $\Diamond C$ and $\Diamond\neg C$; thus,$\top > \neg(\Diamond C \wedge \neg C)$ will be liable to fail at such a state.

The second and third results are blocked because Persistence does not hold in full generality. When A is impersistent, $A > (C > A)$ can fail: updating with A and then updating with C may leave us in one of the substates that now fails to accept A; if so, $A > (C > A)$ may fail to hold.

Finally, an acceptance based semantics will reject CSO for right-nested conditionals, thus avoiding Triviality 4. Here, informally, is a case that illustrates the structure of CSO failures for the acceptance view. Suppose you are contemplating tossing a coin, one which I know is either double-headed or merely heavily tails-biased; in fact, it is double-headed. It seems the following should be true:

(45) If the coin is tossed, it will land heads.

Not because its antecedent epistemically *necessitates* its consequent, but rather just because of how things happen to shake out. We also have:

(46) If the coin lands heads, then the coin will be tossed.

Because of course landing heads requires being tossed in the first place.

Contrary to CSO, the acceptance view says that conditionals of the form *coin tossed* $>$ $(B > C)$ are not in general equivalent to those of the form *coin lands heads* $> (B > C)$. Updating with the information that the coin is tossed is clearly not the same as updating with the information that the coin lands heads. Thus, in conditionals of the second form, but not the first, the nested $B > C$ will hold fixed information about how the toss actually turns out. For example, consider:

(47) If the coin is tossed, then if it is tails-biased, it will land tails.

This is coherent: updating merely with the information that the coin was tossed leaves open that the coin in fact lands tails. On the other hand, consider:

(48) #If the coin lands heads, then if it is tails-biased, it will land tails.

This is not coherent: updating with the coin landing heads is not of course consistent with it landing tails.

6.2 Proof of concept

I now give a particular example of a particular acceptance based semantics, a variably strict domain semantics, where an information state supplies the accessibility relation for the conditional. This theory has some notable precedents in the literature, in particular the path semantics of Goldstein and Santorio (2021) and Santorio (2022).[27] The theory below, however, has the advantage of using only very standard machinery.

Say that a domain frame is a pair $\langle P, f\rangle$ which meets two conditions. First, for some set of worlds W, P is the set of proper world-information state pairs $\langle w, i\rangle$ formed from W; that is, if $p \in P$ then $p = \langle w, i\rangle$ for some $w \in W$ and some $i \subseteq W$ such that $w \in i$. Second,

[27] Another important antecedent is the domain semantics for the conditional in Yalcin (2007). There is, however, one major difference between Yalcin's semantics and mine. Yalcin's semantics essentially says that $A > C$ is true at $\langle w, i\rangle$ just in case the result of updating i to accept $[\![A]\!]$ accepts $[\![B]\!]$. This essentially says that there can be no difference between $A > B$ and $A > \Box B$. This is not a particularly plausible prediction, as Rothschild and Klinedinst (2014) observes. Compare:

(i) If the coin is flipped, it will land heads.

(ii) If the coin is flipped, it must land heads.

If the coin is fair we should take the former to be about 1/2 likely; but the second looks to have 0 probability.

say that f, our selection function, is a function from propositions and points in P to a set of points in P; that is, f is a map from $\mathcal{P}, P$ to P. A domain model adds a valuation function V which assigns to each atomic some subset of W, our set of worlds.

We now define truth in a domain model. Sentences are evaluated for truth at points in P. Booleans only care about the world parameters. Atomics hold at a point just in case the world parameter is in the set of worlds assigned to that atomic by some valuation function; $\wedge$, $\vee$ and $\neg$ have their usual classical semantics. Suppressing reference to a frame and a model we have:

Atomics $\langle w, i\rangle \models p$ iff $w \in V(p)$

Negation. $\langle w, i\rangle \models \neg A$ iff $\langle w, i\rangle \not\models A$

Conjunction. $\langle w, i\rangle \models A \wedge B$ iff $\langle w, i\rangle \models A$ and $\langle w, i\rangle \models B$

Disjunction. $\langle w, i\rangle \models A \vee B$ iff $\langle w, i\rangle \models \neg(\neg A \wedge \neg B)$

I give a variably strict theory semantics for the conditional, stated in terms of the selection function. Intuitively, $f(\mathbf{A}, \langle w, i\rangle)$ outputs the set of closest points to $\langle w, i\rangle$ where $\mathbf{A}$ holds.[28] So, where $[\![A]\!]$ is the set of points $\langle w, i\rangle \in P$ such that $\langle w, i\rangle \models A$, we have:

Variably strict domain semantics. $\langle w, i\rangle \models A > C$ iff for all $\langle w', i'\rangle \in f([\![A]\!], \langle w, i\rangle)$ $\langle w', i'\rangle \models C$

Now, in evaluating a conditional at a point $\langle w, i\rangle$, we want the information parameter to serve as the accessibility relation for the conditional; and we also want the accessibility relation to be updated by successive antecedents. This is achieved by imposing some further constraints on the selection function. Say that $i \models \mathbf{A}$ iff $\langle w, i\rangle \in \mathbf{A}$, for all $w \in i$. Then we have:

Update Constraint. If $\langle w', i'\rangle \in f(\mathbf{A}, \langle w, i\rangle)$ then $i' \in i + \mathbf{A}$, where $i + \mathbf{A} = max\{i' \subseteq i : i' \models A\}$

Non-Vacuity. If $i + \mathbf{A} \neq \{\varnothing\}t$ then $f(\mathbf{A}, \langle w, i\rangle) \neq \varnothing$

The update constraint says that the closest points to $\langle w, i\rangle$ where $\mathbf{A}$ is true are always ones whose information parameter accepts $\mathbf{A}$; specifically for $\langle w', i'\rangle$ to be in $f(\mathbf{A}, \langle w, i\rangle)$, i' must be a maximal $\mathbf{A}$-accepting subset of i. Non-Vacuity says that there is always a closest $\mathbf{A}$-point to $\langle w, i\rangle$ when i can consistently be updated with $\mathbf{A}$. Note that the converse of Non-Vacuity already follows from the Update Constraint.

[28] I use bold variables to range over subsets of P.

The latter three constraints work together provide the familar constraint that the closest **A**-points are themselves **A**-points.

Success. $f(\mathbf{A}, \langle w, i \rangle) \subseteq \mathbf{A}$.

For if $\langle w', i' \rangle \in f(\mathbf{A}, \langle w, i \rangle)$ then $\langle w', i' \rangle$ makes **A** true: for i' must be a maximal, **A**-accepting state; by definition, for every $w'' \in i'$ $\langle w'', i' \rangle \models A$; and $w' \in i'$, since $\langle w', i' \rangle \in P$.

We also impose versions of some standard variably strict constraints. Such theories usually stipulate that if the actual world is an A-world, then it is the closest A world to itself. Here is the analogous constraint in the acceptance semantics:

Minimality. If $\langle w, i' \rangle \in \mathbf{A}$ and $i' \in i + \mathbf{A}$, $\{\langle w, i' \rangle\} \in f(\mathbf{A}, \langle w, i \rangle)$

Roughly, Minimality says, in evaluating a conditional, we shift the *world* that a consequent is evaluated at only if necessary to find a point where the antecedent is true. If **A** is already true at $\langle w, i' \rangle$ and i' is just the result of updating i with **A**, then $\langle w, i' \rangle$ is the closest **A**-point to $\langle w, i \rangle$.[29]

One final constraint is required a restricted form of CSO. Variably strict frameworks also usually impose a constraint like the following: if the closest A-worlds are B-worlds and the closest B-worlds are A-world, then the closest A-worlds are the closest B-worlds. We impose the following analogue. Where **A** is a set of world-information state pairs, say that $\downarrow\mathbf{A}$ is the set of worlds that occur as the world parameter of some element of **A**.[30]

Reciprocity. If $f(\mathbf{A}, \langle w, i \rangle) \subseteq \mathbf{B}$ and $f(\mathbf{B}, \langle w, i \rangle) \subseteq \mathbf{A}$, then $\downarrow f(\mathbf{A}, \langle w, i \rangle) = \downarrow f(\mathbf{B}, \langle w, i \rangle)$

We now turn to the logic of this theory. Validity is preservation of truth at a point: $A_1, ..., A_n \models C$ iff if $\langle w, i \rangle \models A_1$, ..., and $\langle w, i \rangle \models A_n$ then $\langle w, i \rangle \models C$. On this definition, it is easy to see that Identity and Consequent Agglomeration hold. Since Success is derivable from our constraints, Identity is valid. Consequent Agglomeration follows given the set-theoretic entry for the conditional. Persistence holds for any A and C that are themselves persistent:

Persistent Persistence. $A > (C > A)$, when A and C are persistent.

Notice that Boolean Persistence follows from this.

Triviality 1 – 3 all fail in this framework.[31] The argument for Triviality 1 fails when it appeals to the following instance of MOD:

[29] Note that Minimality cannot be formulated in the standard way without clashing with the Update Constraint.
[30] That is, $\downarrow\mathbf{A} = \{w : \langle w, i \rangle \in \mathbf{A}\}$
[31] See Appendix C for countermodels.

$$((A \supset C \wedge \neg(A > C)) > \bot) \supset (B > ((A \supset C) \supset (A > C)))$$

Again we can see this by letting B be $\top$. As I show in Appendix C $(A \supset C \wedge \neg(A > C))$ is an informational contradiction for Boolean A and C. Thus $(A \supset C \wedge \neg(A > C)) > \bot)$ is trivially true: there is no information state which accepts $(A \supset C \wedge \neg(A > C))$; and so there are never any closest points where this antecedent holds. $(\top > ((A \supset C) \supset (A > C))$ is equivalent to $(A \supset C) \supset (A > C)$. And the latter is not valid: simply find a point where $\neg$**A** is false and where the closest **A**-point is a $\neg$**C**-point

The arguments for Triviality 2 and 3 both appeal to similar instances of Persistence, respectively:

(49) $\neg(A > C) > C > \neg(A > C)$

(50) $\neg(A > C) > A > \neg(A > C)$

Neither of these hold on the acceptance semantics because negated conditionals are not persistent: i might accept $\neg(A > C)$ while some subset of i does not.

We can see that negated conditionals are not persistent by observing that, for Boolean A and C, accepting $\neg(A > C)$ requires accepting a possibility claim: it requires accepting that $A \wedge \neg C$ is possible. To accept $\neg(A > C)$ i must contain some A-worlds: otherwise, the result of updating i with A would be empty and so $A > C$ would be trivially true throughout i. Now notice that if i contains only $A \wedge C$-worlds, it also cannot accept $\neg(A > C)$: by Minimality $A > C$ would be true at all such worlds. Putting the two together then, accepting $\neg(A > C)$ requires accepting that $A \wedge \neg C$ is possible.[32]

Now we can see why (49) and (50) both fail. Take (49) first. Updating with $\neg(A > C)$ results in a state that contains an $A \wedge \neg C$-world. But if we then update with C we will no longer have a state that accepts $\neg(A > C)$: the resulting state cannot contain any C-worlds and so in particular cannot contain any $A \wedge \neg C$-worlds. Thus, by the time we have updated with both antecedents, we are no longer in an information state that accepts $\neg(A > C)$. Thus (49) can fail: once we move from the selected $\neg(A > C)$ points to the selected C-points, we may reach a point where $\neg(A > C)$ fails, since $\neg(A > C)$ does not remain accepted after updating with C. The same basic point applies to (50) as well. Once we update with the second antecedent A, we are again in an information state that does not accept $\neg(A > C)$; and so once we move from the selected $\neg(A > C)$-points to the selected A-points, we may reach a point which no longer accepts $\neg(A > C)$.

And what of Triviality 4? Recall CSO.

[32]Notice as well that, for Boolean A, $\neg A > \bot$ is true just in case i contains only A-worlds. Given a domain semantics for *must*, then we obtain Identity of Accessibility for Boolean A. See Appendix C for the proof.

CSO. $((A > B \wedge B > A) \wedge A > C) \supset B > C$

CSO is not valid in full generality: it holds when A, B and C are Boolean, but not when C is a right-nested conditional. The reason is that, even if $A > B$ and $B > A$ are both *true*, updating with A and B may have different effects on the accessibility relation exploited by right-nested conditionals.

We can see this at work by returning to the scenario from §5. Suppose again that the following are very likely (though not certain).

(26) If the Democrat or the third party candidate wins, it will be the third party candidate.

(27) If the Republican or the third party candidate wins, it will be the third party candidate.

Remember that the Republican loses iff the Democrat or the third party wins; and the Democrat loses iff the Republican or the third party wins. Thus CSO, Identity and Consequent Consequence allow us to derive:

(51) If the Republican loses, the Democrat loses; and if the Democrat loses, the Republican loses.

Now if CSO is valid for right-nested conditionals, from (30), entailed by Boolean Persistence, Identity and Consequent Consequence, we can derive the absurd (31).

(30) If the Republican loses, then if third party candidate loses, then the Democrat wins.

(31) If the Democrat loses, then if the third party candidate loses, the Democrat wins.

This last inference fails in our semantics. Suppose that w_3 is the actual world, but we cannot conclusively rule out any of the three candidates. We can thus think of ourselves as being located at $\langle w_3, i \rangle$, where i treats all three candidates as possible victors. CSO fails because, even though (51) holds, the closest point to us where the Republican loses is not the same as the closest point where the Democrat loses. Given (26) the closest point where the Republican loses is $\langle w_3, i \cap \textit{Republican loses} \rangle$; here, the third party wins, worlds where the Democrat wins are also still accessible, but no worlds where the Republican wins are accessible. On the other hand, given (27), the closest point where the Democrat loses is $\langle w_3, i \cap \textit{Democrat loses} \rangle$; here again the third party wins but here the only other accessible worlds are ones where the Republican wins. The truth of (30) only tells us that at $\langle w_3, \textit{Republican loses} \rangle$, it's true that if the third party candidate loses, then the Democrat wins. But since this is not the closest point where the Democrat loses, we are not committed to saying that (31) is also true.

7 Persistence's Revenge

I close with a revenge puzzle. The acceptance picture avoids triviality, at least in part, by denying Persistence holds for impersistent sentences. But even in these cases, Persistence looks required to explain why certain iterated conditionals sound like informational contradictions.[33] I argue the challenge is more general: *all* of the instances of Persistence used to prove Triviality 3 either are plausible in themselves or appear needed to explain something else; every non-trivial approach is faced with some uncomfortable choice. I tentatively suggest that the apparent validity of instances of Persistence should be explained by appeal to *variadic* conditionals.

Epistemic modals can be embedded in conditional antecedents without fuss; so consider sentences with the structure:

(52) If it might be raining, then if it isn't raining, then...

(53) If it might have been raining, then if it wasn't raining, then...

These seem to commit us to an informational contradiction; that is, they seem very close in status to:

(54) If it might be raining and it isn't, then...

(55) If it might have been raining and it wasn't, then...

If we had Persistence for epistemic modal antecedents, this would be easily explained. Given

(56) If it might be raining, then if it isn't raining, then it might be raining.

we then could derive:

(57) If it might be raining, then if it isn't raining, then $\bot$.

Things which entails sentences of this form tend to be defective. But we do not have instances of Persistence like (56): we gave up impersistent instances to avoid triviality.

I think there is a deeper, more general problem here, rather than just an objection to the acceptance based approach.[34] For consider the following instance of Persistence:

$$\neg(\neg A > \bot) > (A > \neg(\neg A > \bot))$$

[33] Thanks to XXX for this observation.

[34] Another symptom of this fact is that, to my knowledge, no existing view explains these data.

Given Identity of Accessibility, we would expect (56) to be equivalent to something of this form. Yet this is in fact exactly one of the three instances of Persistence appealed to in the proof of Triviality 3, the others being:

(58) $\neg(\neg A > \bot) > (\neg(A > \bot) > \neg(\neg A > \bot))$

(59) $A > (\neg A > A)$

Every view has some explaining to do, as a case can be made for all three instances. Given Identity of Accessibility, (58) is equivalent to:

(60) $\Diamond\neg A > (\Diamond A > \Diamond\neg A)$

Claims of this form seem impeccable. Attempts to deny or hedge on them have no plausibility:

(61) If it might be raining, then if might not be raining, then it can't be raining.

(62) If it might be raining, then if it might be not raining, then maybe it must not be raining.

(59) might initially seem more ripe for rejection; after all, its instances do not exactly strike us as felicitous:

(63) # If it is raining, then if it is not raining, then it is raining.

But in fact, Persistence is likely part of the best explanation for why such sentences are infelicitous: given Persistence, Identity and Consequent Agglomeration, we can derive:

(64) If it is raining, then if it is not raining, then $\bot$

I see this as close to a proof that any response must go against some of the data. I think basically we have just two options: either reject Identity of Accessibility; or say that, despite appearances, at least one of (56), (58) and (59) does not in fact correspond to an instance of Persistence. I close by saying how I think the acceptance approach might develop the latter idea, specifically, that claims like (56) are not interpreted as instances of Persistence.

I make two semantic conjectures. The first is that "if" expresses a *variadic function*, a function that can take a *variable* number of inputs. Specfically, "if" can take a variable number of antecedents. To assess such a conditional $(A_1, ..., A_n) > C$, we update with all the antecedents *at once* and assess whether the consequent holds. The entry from §7 is easily adjusted to deliver this:

$\langle w, i\rangle \models (A_1, ..., A_n) > C$ iff for all $\langle w', i'\rangle \in f([\![A_1 \wedge ... \wedge A_n]\!], \langle w, i\rangle) : \langle w', i'\rangle \models C$

On this view, sentences of the form "If A then if B, then C" are structurally ambiguous between a right-nested conditional with one antecedent and a conditional with multiple antecedents:

(65) If A then if B, then C.

a. $A > (B > C)$

b. $(A, B) > C$

On the second kind of reading, the function of the multiple "if"-clauses would be to distinguish arguments that are to be treated as antecedents from the argument serving as the consequent. Thus, in addition to allowing antecedents to be updated successively, simple "right-nested" conditionals can also be understood as collecting together the various antecedents and updating with them all at once.

So far, this only says that sentences of the form "if A then if B then C" are ambiguous. This does not yet tell us why sentences of the form (56) are defective. My second conjecture is that there is a ban on impersistent structures like (56). One way to implement this is as a definedness constraint. Add to our set of parameters a premise set Γ and say that, in addition to updating the information state, conditionals also add their antecedent to the premise set:

Variadic Conditional. $\langle w, i\rangle, \Gamma \models (A_1, ..., A_n) > C$ iff for all $\langle w', i'\rangle \in f([\![A_1 \wedge ... \wedge A_n]\!], \langle w, i\rangle) : \langle w', i'\rangle, \Gamma \cup \{A_1, ..., A_n\} \models C$

We can then add the constraint that $\langle w', i'\rangle, \Gamma \models A$ only if i' accepts all the members of Γ:

Definedness constraint. $\langle w', i'\rangle, \Gamma \models A$ only if $i' \models \gamma$, for all $\gamma \in \Gamma$.

This now predicts that the normal right-nested interpretation of sentence of the form $\Diamond A > \neg A > C$ will be undefined. The successive antecedents will do two things: they will update the information state first with $\Diamond A$ and then with $\neg A$; and they will add $\Diamond A$ and then $\neg A$ to the premise set. At this point, we will have the kind of mismatch ruled out by the definedness constraint: the information state will not accept all members of Γ: in particular, it will not accept $\Diamond A$.

This suggestion raises a number of questions. First, is the kind of variadic account proposed above consistent with compositionality? I am inclined to think that on a reasonable understanding of compositionality, it should be: given that such functions are common in various programming languages, I am skeptical that there can be especially good principled reasons for prohibiting them in natural languages. But I will also note that it is also possible to implement the above idea in a more orthodox semantic implementation. I leave the details to a footnote.[35]

A more pressing question is whether postulating variadic functions undermines the motivation for validating any form of Persistence. Could we not simply drop Persistence entirely for true right-nested structures and instead explain its appeal with these variadic conditionals?

The answer, I think, is no. One important motivation for Persistence was restricted readings of modals. Recall:

(5) If we are near Venice Beach, then we must not be in Kansas.

We saw that, given Persistence and Identity of Accessibility, we can explain such readings via the validity of $A > \Box A$, for non-modal A. If we give up Persistence entirely in favour of some syntactic explanation, this motivation is lost: it is not possible to construe (5) as a variadic conditional.

A similar case can be made using negated right-nested conditionals. Consider

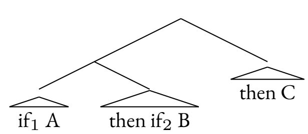
(66) If it's either raining or snowing, then it is still not the case if it's sunny, it is snowing.

This cannot be construed variadically, not without taking serious liberties with syntax. But here too we need to be able to appeal to Persistence: from (66) we can infer:

(67) If it's either raining or snowing, then if it's sunny it might be raining.

If we do not have Persistence for true right-nested Boolean conditionals, it's unclear how such inferences would go through.

[35]First, we postulate two possible syntactic structures conditionals with the surface form "if A, then if B then C". In addition to the more familiar right-nested structure, we postulate they can be interpreted with the following structure:



Note that there is in fact precedent for positing structures like these: Khoo (2021b,a) and Starr (2014) postulate such structures to account for conditionals with coordinated antecedents.

Now two separate entries for "if$_1$" and "if$_2$": say that $[\![if_2]\!]$ takes a proposition and forms the singleton of that proposition; say that $[\![if_1]\!]$ takes a set of propositions $\{A_2, \ldots, A_n\}$, a further proposition A_1 and the consequent C and maps them to the intension for the variadic conditional. To cover the case where we have more than two antecedents, we can add the following semantic rule: if a node α has as its daughters β and γ and if the semantic values of both β and γ are sets of propositions, then $[\![\alpha]\!] = [\![\beta]\!] \cup [\![\gamma]\!]$.

Why would natural language allow for the possibility of impersistence, only to wipe it out again via these definedness conditions? While such a question is troubling, I suspect that, given my arguments above, everyone will be faced with some version of this question. For after all, everyone will have to say that at least one of the three instances of Persistence, (56), (58), or (59) fails. But none are obviously invalid when we look at natural language examples. No matter what route we take, natural language has some expressive possibilities that it refuses to manifest.

A Import-Export, Persistence, MOD and Contradiction

Fact 1. In PC + Identity, Consequent Agglomeration and MOD, Import-Export entails Persistence but not vice versa.

Proof. An official proof that, given Identity, CA and MOD, I-E entails Persistence can be easily extracted from §3.

To show the converse fails we give a model theoretic argument. A Stalnaker frame contains a set of worlds and a selection function f; a model adds a valuation to a frame; and the semantics for the conditional is given using the selection function: $V(A > C, w) = 1$ iff $f([\![A]\!], w) \subseteq [\![C]\!]$).[36] All of the rules of PC are sound on Stalnaker frames, as is Consequent Agglomeration; see Chellas (1975). Thus on a frame that validates Identity, MOD and Persistence, any theorem of the system from §2 will be valid. We thus give a frame where Identity, CA, MOD and Persistence are valid but Import-Export is not.

- $W = \{w_1, w_2, w_3\}$
- Selection function:
  - For all $\mathbb{A} \subseteq W$, $f(\mathbb{A}, w_1) = \{w_1\}$ if $w_1 \in \mathbb{A}$; otherwise $= \varnothing$
  - For all $\mathbb{A} \subseteq W$, $f(\mathbb{A}, w_2) = \{w_2\}$ if $w_2 \in \mathbb{A}$; otherwise $= \varnothing$
  - $f(W, w_3) = w_2$; $f(\{w_1, w_2\}, w_3) = \{w_1\}$; $f(\{w_1, w_3\}, w_3) = \{w_1\}$; $f(\{w_2, w_3\}, w_3) = w_2$; $f(\{w_1\}, w_3) = \{w_1\}$; $f(\{w_2\}, w_3) = \{w_2\}$; $f(\{w_3\}, w_3) = \varnothing$.

The Stalnaker semantics trivially validates Consequent Agglomeration. On this particular frame, Identity is valid: by construction, for any $\mathbb{A}$, $f(\mathbb{A}, w) \subseteq \mathbb{A}$.

MOD is valid. Otherwise there would be some V and some w such that $V(A > \bot, w) = 1$ but $V(C > \neg A, w) = 0$. This cannot be w_1: if $f([\![A]\!], w_1) = \varnothing$ then by construction

[36] Another word on notation: I use $\mathbb{A}$ as a variable over sets of worlds. Given a particular model $\mathcal{M}$, $[\![A]\!]^{\mathcal{M}}$ is the set of worlds where A is true in $\mathcal{M}$; and I supress the superscript when the intended model is obvious.

$w_1 \notin [\![A]\!]$; but if also $f([\![C]\!], w_1) \neq \varnothing$ then $w_1 \notin [\![\neg A]\!]$ and so $w_1 \in [\![A]\!]$. Similarly, it cannot be w_2. Now take w_3. Since $f(\mathbb{A}, w_3) = \varnothing$ iff $\mathbb{A} = \{w_3\}$, if $V(A > \bot, w_3) = 1$ then $[\![A]\!] = \{w_3\}$. Now if $V(C > \neg A, w_3) = 0$ then $[\![C]\!] \neq \varnothing$. But in the frame above, when $[\![C]\!] \neq \varnothing$ then $f([\![C]\!], w_3) \subseteq \{w_1, w_2\} = [\![\neg A]\!]$. So in fact $V(C > \neg A, w_3) = 1$.

Now suppose Persistence fails. Then some valuation must make $A > (C > A)$ false at some world. Now Persistence cannot fail for w_1 and w_2. For if $A > (C > A)$ fails at w_1, then $f([\![A]\!], w_1) \neq \varnothing$ and so by construction $f([\![A]\!], w_1) = \{w_1\}$. So then $f([\![C]\!], f([\![A]\!], w_1)) = f([\![C]\!], w_1)$ which is either $\{w_1\}$ or $\varnothing$ and so a subset of $[\![A]\!]$; contradiction. The same argument shows that Persistence cannot fail at w_2. So suppose $A > (C > A)$ fails at w_3. Then $[\![A]\!] \neq \{w_3\}$. So $f([\![A]\!], w_3)$ is either $\{w_2\}$ or $\{w_3\}$. But for $1 \leq i \leq 2$ $f([\![C]\!], w_i) = \{w_i\}$ or $= \varnothing$ and thus $f([\![C]\!], w_i) \subseteq [\![A]\!]$. So in fact $f([\![C]\!], f([\![A]\!], w_3)) \subseteq [\![A]\!]$; contradiction.

Import-Export, however, is not valid. Set $[\![p]\!] = W$ and $[\![q]\!] = \{w_1, w_2\}$ and $[\![r]\!] = \{w_2\}$. At w_3 Import-Export fails. $p > q > r$ is true: $f([\![q]\!], f([\![p]\!], w_3)) = \{w_2\} = [\![r]\!]$. But $(p \wedge q) > r$ is false: $f([\![p \wedge q]\!], w_3) = \{w_1\} \nsubseteq [\![r]\!]$.

Fact 2. Modus Ponens is not a theorem of PC + Persistence, Identity, CA and MOD.

Proof. The frame from Fact 1 demonstrates this. Weak Centering fails at w_3: for instance, $f(\{w_1, w_3\}, w_3)$ is w_1, even though $w_3 \in \{w_1, w_3\}$. As is well-known, Weak Centering charactersises Modus Ponens in these frames.

The failure of Weak Centering is in fact essential to the proof of Fact 1. Given Modus Ponens, we can recreate Gibbard's triviality result. From Persistence we already have $C > (A > C)$ and $\neg A > (A > C)$ is easily derived given Persistence and the logic in §2. The rest of the proof is the same as Gibbard's. Since the material conditional obeys Import-Export, we cannot prove Fact 1 in any frames with Weak Centering.

Fact 3. Contradiction is not a theorem of PC + Import-Export, Identity, CA and MOD.

Proof. Take a simple Stalnaker frame where for every A, $f(A, w) = \varnothing$. Import-Export, Identity, and MOD are all trivially valid on this frame: for any A and C, $A > C$ is valid on this frame. (As before, CA is valid on all Stalnaker frames.) But Ad Falsum is not valid: for some arbitrary world w set $[\![p]\!] = \{w\}$. $p > q$ and $p > \neg q$ are both trivially true at w; $\neg p$ is false.

B Proof of Triviality Results

Triviality 1. $B > ((A \supset C) \supset (A > C))$

Proof.

1. $((A \supset C) \wedge \neg(A > C)) > ((A \supset C) \wedge \neg(A > C))$ (Identity)

2. $((A \supset C) \wedge \neg(A > C)) > \neg(A > C)$ (CA,1)

3. $((A \supset C) \wedge \neg(A > C)) > (A > ((A \supset C) \wedge \neg(A > C)))$ (Persistence)

4. $((A \supset C) \wedge \neg(A > C)) > (A > (A \supset C))$ (CA, 3)

5. $((A \supset C) \wedge \neg(A > C)) > (A > A)$ (CA, Identity)

6. $((A \supset C) \wedge \neg(A > C)) > (A > C)$ (CA, 4,5)

7. $((A \supset C) \wedge \neg(A > C)) > \bot$ (CA, 2,6)

8. $B > ((A \supset C) \supset (A > C))$ (MOD, CA, 8)

Note the following easy to prove lemma:

Lemma. If $\vdash C$ then $\vdash A > C$

Now recall:

Triviality 2. $\neg(A > C) > B > \neg C$

Proof.

1. $C > (A > C)$ (Persistence)

2. $\neg(A > C) > \neg(A > C)$ (Identity)

3. $\neg(A > C) > (C > (A > C))$ (2, lemma)

4. $\neg(A > C) > (C > \neg(A > C))$ (Persistence)

5. $\neg(A > C) > (C > \bot)$ (Consequent Agglomeration, 3,4)

6. $\neg(A > C) > (B > \neg C)$ (MOD, 4)

Note an easy to prove theorem:

Theorem $*$. $\neg(A > C) \supset \neg(A > \bot)$

Now recall:

Triviality 3. $\neg(A > B) > \neg(\neg A > C) > \bot$

Proof:

1. $\neg(A > B) > (\neg(\neg A > C) > \neg(A > B))$ Persistence
2. $\neg(A > B) > (\neg(\neg A > C) > \neg(A > \bot))$ 1, theorem $*$, Consequent Agglomeration
3. $\neg(\neg A > C) > (A > \neg(\neg A > C))$ Persistence
4. $\neg(\neg A > C) > (A > \neg(\neg A > \bot))$ 3, theorem $*$, Consequent Agglomeration
5. $A > (\neg A > A)$ Persistence
6. $A > (\neg A > \neg A)$ Lemma
7. $A > (\neg A > \bot)$ 5,6, Consequent Agglomeration
8. $\neg(\neg A > C) > (A > (\neg A > \bot))$ 7, lemma
9. $\neg(\neg A > C) > (A > \bot)$ 4,8, Consequent Agglomeration
10. $\neg(A > B) > (\neg(\neg A > C) > (A > \bot))$ 9, lemma
11. $\neg(A > B) > (\neg(\neg A > C) > \bot)$ 2,10, Consequent Agglomeration

Triviality 4. $(A \vee B) > B, (B \vee C) > B \vdash (A \vee B) > \neg B > \bot$, given Identity, CA, CSO and Boolean Persistence we can prove, for pairwise inconsistent Boolean A, B, C:

Proof.

1. $(A \vee B) > B$ (assumption)
2. $(B \vee C) > B$ (assumption)
3. $B > (A \vee B)$ (Identity, CC)
4. $B > (B \vee C)$ (Identity, CC)
5. $(A \vee B) > (B \vee C)$ (1,3,4, CSO)
6. $(B \vee C) > (A \vee B)$ (2,3,4, CSO)
7. $(A \vee B) > (\neg B > (A \vee B))$ (Boolean Persistence)
8. $(A \vee B) > (\neg B > \neg B)$ (Identity, Consequent Consequence)

9. $(A \vee B) > (\neg B > A)$ (7,8, Consequent Agglomeration)

10. $(B \vee C) > (\neg B > (B \vee C))$ (Boolean Persistence)

11. $(B \vee C) > (\neg B > \neg B)$ (Identity, Consequent Consequence)

12. $(B \vee C) > (\neg B > C)$ (10, 11, Consequent Agglomeration)

13. $(A \vee B) > (\neg B > C)$ (5,6,12, CSO)

14. $(A \vee B) > (\neg B > \bot)$ (9, 13, Consequent Agglomeration)

C Proofs of claims in §6

Fact 1. Identity is valid.
Proof. First recall that $i \models A$ iff for all $w \in i$ $\langle w, i\rangle \models A$. This is trivial when $f(\llbracket A \rrbracket, \langle w, i\rangle)$ is empty; so assume otherwise. The Update Constraint tells us that if $\langle w', i'\rangle \in f(\llbracket A \rrbracket, \langle w, i\rangle)$ then $i' \models A$. Moreover, since $\langle w', i'\rangle \in P$, $w' \in i'$. Thus $\langle w', i'\rangle \models A$.

Fact 2. Consequent Agglomeration is valid.
Proof. Routine.

Fact 3. When A is Boolean, $i + A = i \cap \llbracket A \rrbracket^i$, where $\llbracket A \rrbracket^i = \{w : \langle w, i\rangle \in \llbracket A \rrbracket\}$.
Proof. Routine induction on complexity.

Say that $\langle w, i\rangle \models \Box A$ iff for all $w' \in i$ $\langle w, i'\rangle \models A$; and define $\Diamond A$ as $\neg \Box \neg A$.

Fact 4. Identity of Accessibility holds for Boolean A.
Proof. First suppose $\neg A > \bot$ holds at $\langle w, i\rangle$. Then $f(\llbracket \neg A \rrbracket, \langle w, i\rangle)$ is empty. Thus by Non-Vacuity $i + \neg A$ is empty. By Fact 3, $i \cap \llbracket \neg A \rrbracket^i$ is thus empty and so for all $w' \in i : \langle w', i\rangle \models A$. Thus $\langle w, i\rangle \models \Box A$. Now suppose $\langle w, i\rangle \models \Box A$. Thus $w' \in i : \langle w', i\rangle \models A$. Thus $i + \neg A$ is empty. The Update Constraint says that if $\langle w', i'\rangle \in f(\llbracket \neg A \rrbracket, \langle w, i\rangle)$ then $i' = i + \neg A$; but, since $\langle w', i'\rangle \in P$ and so $w' \in i'$, if $i + \neg A$ is empty then $f(\llbracket \neg A \rrbracket, \langle w, i\rangle)$ must also be empty. So $\neg A > \bot$ holds at $\langle w, i\rangle$.

Fact 4. $\neg(A > C)$ is not persistent for Boolean A and C.
Proof. Note that $i \models \neg(A > C)$ only if $i \models \Diamond A$. Otherwise $i + A$ contains only the empty set and thus by the Update Constraint, $f(\llbracket A \rrbracket, \langle w, i\rangle)$ would be empty for all $w \in i$. But now since $\Diamond A$ is clearly not persistent, neither is $\neg(A > C)$.

Fact 4. Persistence is invalid.
Proof. $\langle w, i\rangle \not\models \neg(p > q) > (\neg p > \neg(p > q))$, when i contains some p-worlds. $\neg(p > q) > (\neg p > \neg(p > q))$ holds at $\langle w, i\rangle$ iff for all $\langle w', i'\rangle \in f(\llbracket \neg(p > q) \rrbracket, \langle w, i\rangle)$ for all $\langle w'', i''\rangle \in$

$f(\llbracket p \rrbracket, \langle w', i' \rangle\ \langle w'', i'' \rangle \models \neg(p > q)$. $\langle w'', i'' \rangle \in f(\llbracket p \rrbracket, \langle w', i' \rangle\ \langle w'', i'' \rangle \models \neg(p > q)$ iff for some $\langle w''', i''' \rangle \in f(\llbracket p \rrbracket, \langle w'', i'' \rangle)\ \langle w''', i''' \rangle \models \neg q$. But note that $i'' = i' + \neg p$ and $i''' = i'' + p$ so i''' is empty. So $f(\llbracket p \rrbracket, \langle w'', i'' \rangle)$ is in fact empty.

Fact 5. Persistent Persistence is valid.

Proof. Recall that A is persistent iff when $i \models A$ and $i' \subseteq A$ then $i' \models A$. $A > (C > A)$ holds at $\langle w, i \rangle$ iff for all $\langle w', i' \rangle \in f(\llbracket A \rrbracket, \langle w, i \rangle)$: for all $\langle w'', i'' \rangle \in f(\llbracket C \rrbracket, \langle w', i' \rangle)$ $\langle w'', i'' \rangle \models A$. Pick an arbitrary $\langle w', i' \rangle \in f(\llbracket A \rrbracket, \langle w, i \rangle)$ and an arbitrary $\langle w'', i'' \rangle \in f(\llbracket C \rrbracket, \langle w', i' \rangle)\ \langle w'', i'' \rangle \models A$. First note that $i' \in i + A$ and so $i' \models A$. Now $i'' = i' + C$. Since A is persistent $i'' \models A$ also. Since $\langle w'', i'' \rangle \in P$, $w'' \in i''$. So $\langle w'', i'' \rangle \models A$.

Fact 6. $i \models (A \supset C)$ just in case $i \models (A > C)$, for Boolean A, C.

Proof. Suppose that $i \models A \supset C$. Then $i \cap \llbracket A \rrbracket^i \cap \llbracket \neg C \rrbracket^i$ is empty. If $w \in i \cap \llbracket A \rrbracket^i$ then $\langle w, i \rangle \models A \wedge C$ and so $\langle w, i \rangle \models A > C$, by Minimality. Suppose then $w \in i \cap \llbracket \neg A \rrbracket^i$. Let $\langle w', i' \rangle \in f(\llbracket A \rrbracket, \langle w, i \rangle)$: $i' = i + A$ and $w' \in i + A$. Thus $\langle w' i' \rangle \models C$. Now suppose that $i \not\models A \supset C$. Then for some $w \in i$ $w \in \llbracket A \rrbracket \cap \llbracket \neg C \rrbracket$. Then by Minimality $A > C$ fails at $\langle w, i \rangle$.

Fact 7. MOD is invalid.

Proof. $((p \supset q) \wedge \neg(p > q)) > \bot$ can be true, even while $r > ((p \supset q) \supset (p > q))$ fails at some $\langle w, i \rangle$. It is easy to see the former is trivially true. Since no $i \models (p \supset q) \wedge \neg(p > q)$, $i + (p \supset q) \wedge \neg(p > q)$ is empty and thus $f(\llbracket (p \supset q) \wedge \neg(p > q) \rrbracket, \langle w, i \rangle)$ is empty. Now take a model with three worlds w_1, w_2 and w_3. Let r be true only at w_2 and w_3 and p be true only at w_3; q is true nowhere. Notice that necessarily $f(\llbracket r \rrbracket, \langle w_1, \top \rangle) = \{\langle w_2, \llbracket r \rrbracket \rangle, \langle w_3, \llbracket r \rrbracket \rangle\rangle\}$. $p \supset q$ is true at $\langle w_2, \llbracket r \rrbracket \rangle$. But $p > q$ is false: $f(\llbracket p \rrbracket, \langle w_2, \llbracket r \rrbracket \rangle) = \langle w_3, \{w_3\} \rangle$.

Fact 8. CSO holds for Boolean A, B, C.

Proof. Suppose that A, B, C are Boolean and that $A > B$, $B > A$, and $A > C$ hold at $\langle w, i \rangle$. Thus $\downarrow f(\llbracket A \rrbracket, \langle w, i \rangle) \subseteq \llbracket B \rrbracket^i$ and $\downarrow f(\llbracket B \rrbracket, \langle w, i \rangle) \subseteq \llbracket A \rrbracket^i$. By Reciprocity $\downarrow f(\llbracket A \rrbracket, \langle w, i \rangle) = \downarrow f(\llbracket B \rrbracket, \langle w, i \rangle)$. Since C is Boolean, for all $\langle w', i' \rangle \in f(\llbracket A \rrbracket, \langle w, i \rangle)$: $\langle w', i' \rangle \models C$ iff $\langle w', i'' \rangle \in f(\llbracket B \rrbracket, \langle w, i \rangle) : \langle w', i'' \rangle \models C$.

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